# Power amplifiers <br> Efficiencies • matching • "rms" power 

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All amplifiers are power amplifiers in that the power delivered to the load is greater than that drawn from the source. Few are power amplifiers in the same sense that an operational amplifier with feedback may be said to be a voltage amplifier or a current amplifier. Thus in Fig. 1 the load voltage is defined for a given input signal and the load power is proportional to the conductance of $R_{L}$. For Fig. 2 feedback defines the load current while the corresponding power developed in the load is proportional to the resistance of $R_{L}$.

This suggests that as many power amplifiers use shunt-derived feedback to define their output characteristics, they could properly be regarded as voltage amplifiers which just happen to be capable of delivering large powers to a load of sufficiently low resistance. Operating these amplifiers from a constant supply voltage in the usual way fixes an upper limit on the load voltage. Practical imperfections in the transistors together with current limiting resistors or other protective circuitry reduce this upper limit but still leave the peak output voltage broadly proportional to the supply.

Output power depends equally on the maximum current that can be supplied to the load. The mean value of this power over a complete cycle for a sinusoidal output voltage and resistance load is $\hat{V} / 2$, where $\hat{V}$ and $\hat{I}$ are the peak instantaneous values of voltage and current, and as $\hat{V}=\hat{I} R_{L}$ and $V_{r m s}=\hat{V} / 2$, alternative expressions are $V_{r m s} \cdot I_{r m s}=\hat{V}^{2} / 2 R_{L}=I^{2} R_{L} / 2=V_{r m s}^{2} / R_{L}=$ $I_{r m s}^{2 m s} R_{L}$.

At this point, you might be asking: "what about r.m.s. power?". This, unfortunately, is on a level with the equivalent enquiry after the well-being of the workers. It can be answered in various ways none of which are useful. To interpret it properly it must be realized that, while the power developed in the load varies from instant to instant, it is the mean or average value of that power that determines, for example, the loudness of the sound produced by a given loudspeaker. The r.m.s. value of the power can be defined mathematically in the same manner as the r.m.s. value of the voltage, but it has no comparable physical significance, i.e. it would require the instantaneous power to be squared, integrated to obtain the mean value of the "power squared", and then the
square root taken of the result. The confusion arose because mean power happens to equal the product of r.m.s. voltage and r.m.s. current for certain specified conditions which commonly occur. Hence the "r.m.s." term has become firmly attached to the power measurement itself particularly in the advertising for audio equipment. It should be detached.

While the voltage term in the output power is fixed by the supply voltage, the current term is a property of the amplifier. Consider first the amplifiers based on a single transistor as shown in Fig. 3. In each case the transistor is assumed to be dealing with an a.c. signal which has equal positive and negative magnitudes, e.g. a sine wave. Thus the transistor must be biased to some quiescent voltage/current setting which will allow positive and negative-going output swings. If distortion is to be avoided the transistor must remain conducting throughout the cycle, i.e. neither the current nor the p.d. across the transistor shall fall to zero. This mode of operation, class A, may be

$P_{o} \propto{ }^{1 / R_{L}}$
Figs. 1 and 2. In a voltage amplifier and a current amplifier, load voltage or current is defined for a given input signal and load power is proportional to the conductance of $R_{L}$ (Fig. 1 above) or the resistance of $R_{L}$ (Fig. 2 below), suggesting that power amplifiers using shunt-derived feedback could be looked on as voltage amplifiers, but capable of handling large powers.

defined in terms of the "angle of conduction", being the full $360^{\circ}$ of the cycle. In class B each device conducts for precisely $180^{\circ}$ or half the cycle and in class C conduction is for $<180^{\circ}$.
In Fig. 3 (a) if the direct current is permitted to flow in the load, equal positive and negative excursions occur for a voltage across the transistor equal to half the supply voltage (assuming an ideal transistor). Thus the peak of the a.c. component of load voltage is $\hat{V}=V / 2$. Hence the a.c. power in the load is $(V / 2)^{2} / 2 R_{L}=V^{2} / 8 R_{L}$. In the absence of signal the current drawn from the supply is $V / 2 R_{L}$ giving a supply power $V^{2} / 2 R_{L}$. This shows an efficiency of 0.25 . The quiescent power splits $50 / 50$ between transistor and load resistance.

It is possible to do still worse. In Fig. 3 (b) the load is capacitively coupled to the amplifier to eliminate the direct current in the load. A collector resistor is still required to allow the flow of current in the transistor, establishing the quiescent conditions for class A operation. Now the total a.c. power is split between $R$ and $R_{L}$ and the maximum efficiency is reduced to 0.125 .

The situation can be retrieved if the collector resistor can be replaced by some constant-current stage as in Fig. 3 (c). The positive peak current in the load can then equal the quiescent current even when the collector approaches the supply voltage (assuming a constant-current stage that can function with a p.d. falling towards zero). Hence the load can have a maximum current swing simultaneously with a maximum voltage swing. In Fig. 3 (b) when the transistor current falls to zero, $R$ and $R_{L}$ are effectively in series and the p.d. across $R$ limits that across $R_{L}$.
The constant-current stage may consist of an inductor whose reactance is high compared to the resistance of the load at all frequencies of interest; an ideal transformer that also allows the use of arbitrary load resistance by proper choice of turns ratio; a transistor biased to deliver a constant current. The disadvantage of (d) and (e) is that the amplitude-frequency response is limited unless bulky and expensive inductor/transformers are available. They do offer the possibility of higher efficiency than any of the other circuits. For example, Fig. 3 (f) allows the peak current in the load to equal the quiescent current, and the peak voltage to equal the transistor quiescent voltage,
i.e. half the supply voltage for the best operating conditions. Thus load power is $\left(I_{d c} V / 2\right) / 2$ while supply power is $V . I_{d c}$, giving an efficiency of 0.25 bringing the efficiency back to the level of Fig. 3 (a) but with the d.c. component removed from the load. In Fig. 3 (d), the peak current in the load is still equal to the quiescent current, but the inductance allows the collector voltage to swing positive with respect to the supply line as the transistor current falls-a load peak voltage equal to the supply voltage being available. The a.c. power in the load then becomes $V_{d c} / 2$ for the same supply power $V_{d c}$, and maximum efficiency is 0.5 . This is the highest efficiency possible in class A and the transformer-coupled circuit of Fig. 3 (e) has the same capabilities. It is common for practical circuits using small transformers to have efficiencies in the region of 0.25 to 0.4 .

The low efficiencies attained by these single-device circuits lead to the investigation of multiple transistor circuits. Simply operating transistors in parallel may increase the quiescent power they can dissipate and hence the available output power, but the method offers only second-arder improvements in efficiency by reduction of saturation voltage etc. Before turning to other classes of amplifier, consider the natural extensions that are possible of the circuit of Fig. 3 (f). Replace the transistor used as a current source by one receiving a signal as in Figs 4 (a) to (d). The signals to the two transistors depend on the configuration used, but the aim in each case is to cause one transistor to decrease its current by the same amount as the other increases it, still assuring class A operation for each transistor individually. In this way the peak current in the load may equal twice the
(a)

(e)



Fig. 3. In a resistive-load class A amplifier (a) efficiency is only $25 \%$; with a capacitatively-coupled load efficiency could be $12 \frac{1}{2} \%$ for equal values of resistor shown (b). Replacing collector resistor by a constant-current circuit (c) means the peak load current can equal the quiescent current. Examples of constant-current circuits are a simple inductor (d) or transformer (e) both giving maximum possible efficiencies of $50 \%$, or at ( $f$ ) using an additional transistor, efficiency $25 \%$.
circuit quiescent current: at the point when the current in one transistor reaches zero the other has doubled. The peak load voltage remains at half the supply voltage when the output is biased to the supply mid-point for maximum undistorted output.
So far the term "matching" has been avoided. For low-level amplifiers, input impedance is frequently matched to that of the source; this is the condition for minimum noise. To maximize the power gains of following stages the output and input impedances may be matched, i.e. made equal. This remains the practice in r.f. amplifiers, but at lower frequencies deliberate mismatch is more common as it allows for better control over the gain.

A fallacy that is based on the experience with these low-level stages, and derives from the maximum power-transfer theorem is often extended to power amplifiers. For each of the class A stages described earlier there is a value of load resistance that maximizes the output power without clipping the waveform peaks. As shown for each individual case there are separate limits for both peak voltage and peak current, and the optimum load will have a resistance given by the ratio of these peaks. This load resistance has nothing to do with the output resistance of the transistor.

Consider Fig. 5 which represents the operation of Fig. 3 (d). Draw the load line representing the load resistance through the quiescent point; the maximum output power without distortion is achieved for the slope giving symmetrical voltage and current swings in the positive and negative directions. Such a line intersects the $V, I$ axes at $2 V_{c}$ and $2 I_{c}$ respectively and the slope of the line is the same as that joining the $V_{c}, I_{c}$ points on the axes. This optimum load is thus confirmed as depending on the quiescent conditions with the slope of the transistor characteristics (the true output resistance) playing no part. Life is rarely that simple in practice, and the results are modified by saturation effects as well as by the various non-linearities, but not sufficiently to disturb the principle, which applies equally to the circuits of Fig. 4.
Hence for Fig. 4 (c) when used as a class A amplifier the quiescent current $\left(I_{s}\right)$ may be calculated from the supply voltage $\left(V_{s}\right)$ and the intended load resistance. Peak voltage in load is $V_{s} / 2$; peak current in load is $2 I_{s}$; therefore optimum load resistance is $V_{s} / 4 I_{s}$. Resulting mean load power is $\left(V_{s} / 2\right) 2 I_{s} / 2=$ $V_{s} I_{s} / 2$, corresponding to an efficiency of 0.5 for ideal transistors.

(a)

(b)


Fig. 4. Using a second for signal handling enables the peak load current to equal twice the quiescent current, with efficiencies of up to $50 \%$.

These circuits are not restricted to class A operation. Indeed they are more commonly used as push-pull class-B amplifiers in which the bias network (not shown) is adjusted to bring both transistors to the edge of conduction. Each transistor then conducts during one half-cycle, there being no quiescent current. There is no comparable limit to the peak current that may be provided; class B simply demands that conduction takes place in a device for $180^{\circ}$ in the cycle. A limit will be imposed in any particular design by the current/power limitations of the transistors/power supply. In principle any basic design for a class $B$ power amplifier using configurations such as those of Fig. 3 (c) and (d) may be extended to higher current levels by replacing the output transistors with Darlington pairs, complementary pairs etc. Thus 100 W and 100 mW amplifiers may be surprisingly similar in configuration. At high power levels the importance of protection and of maintaining stable bias leads to the addition of circuits monitoring and/or controlling the current in the output stage.

To minimize the distortion due to device non-linearity at low currents (cross-over distortion) the bias networks are set to provide some quiescent current, setting the operation intermediate between true class B and class A -often called class AB and further subdivided into $A B_{1}, \mathrm{AB}_{2}$ according to the fraction of the cycle for which each device is non-conducting. The design of low-distortion power amplifiers is a highly specialized subject that will warrant separate treatment in a later series though outline design procedures and practical examples of simple and economical circuits are given in this series of Circards.

Quiescent power in class B is zero. Maximum output power with ideal transistors (Fig. 6) is $\hat{V}^{2} / 2 R_{L}$ where $\hat{V}$ is $V_{s} / 2$. Therefore $P_{\mathrm{L} \text { max }}$ is $V_{s}^{2} / 8 R_{L}$.

Under these conditions, the mean current drawn from the supply is $V_{s} / 2 \pi R_{L}$. This is because the current is drawn from the supply only, during the positive half-cycle; the negative half-cycle results in charge being withdrawn from the large coupling capacitor, which charge is restored during the next positive half-cycle. The mean power drawn from the supply is $V_{s} I_{d c}=V_{s}^{2} / 2 \pi R_{L}$ and the corresponding efficiency is

$$
\frac{P_{L}}{P_{s}}=\frac{V_{s}^{2}}{8 R_{L}} \cdot \frac{2 \pi R_{L}}{V_{s}^{2}}=\frac{\pi}{4}
$$

or $78.5 \%$.
As the load power is proportional to the square of the output voltage while the supply power is proportional to the voltage it follows that efficiency is proportional to output voltage. It is also true that at some intermediate level of output, the load power having fallen faster than the supply power, the power in the transistors passes through a maximum. For sine wave drive the maximum dissipation in each transistor is at an output voltage where $\hat{V}$ is $V_{s} / \pi$, and the dissipation is then about one fifth of the maximum output power derived above, i.e. a 10W amplifier could theoretically be constructed using a pair of transistors with power ratings of only 2 W each.


Fig. 5. Optimum load for maximum signal handling is represented by load line cutting axes at $2 V_{c}, 2 I_{c}$, for Fig. 3 (d) or (e) and depends on the quiescent condition and not transistor output resistance. In general optimum load can be calculated from peak voltage/peak current.


Fig. 6. In this typical class B stage, text shows maximum efficiency to be $78.5 \%$.

Class C amplifiers are normally restricted to tuned amplifier systems, conduction taking place for only a small part of a cycle, and the recovery of a sinusoidal waveform then demands a high- $Q$ circuit. The exception is the power-control circuits such as controlled rectifier and triac circuits in which only the mean value of voltage/ current/power is of interest and waveform shape is non-critical. These differ from conventional class $C$ circuits in that the conduction has a controlled starting angle but always finishes near the end of a half-cycle; class C r.f. amplifiers are biased such that the conduction angle is symmetrical about the peak of the drive waveform.

A further difference is that the power control devices are operated as nearly as possible as perfect switches, while at the high frequencies normally associated with class $\mathbf{C}$ stages, a very detailed design procedure is required to cope with transistor parameters. This will normally include complex conjugate matching to source and and load, to optimise performance. Efficiency can exceed that for class B, though power losses in the passive components involved in the tuning/matching processes are inevitable. A further application of class $\mathbf{C}$ power amplifiers is in frequency multiplication where the output circuit is tuned to a harmonic of the input frequency. These aspects are germane to more detailed later studies of r.f. circuits.

Class D is the generic term for switching circuits in which the active devices are switched multiply in and out of conduction during a single cycle of the input signal. They
are also power realizations of pulse modulation systems, the theory for which can be used to determine the spectral components of the output. As one example, circuits such as those of Fig. 4 may have their signal drive replaced by high-speed pulse waveforms whose widths are modulated by the received signal. If the load is fed via an $L C$ filter, the pulse frequency terms can be removed and the output transferred to the load is proportional to the signal.
For ideal transistors the switching process allows for zero power dissipation; at all times either the transistor p.d. or its current tend to zero. If the unwanted terms are to be well outside the band of frequencies that it is required to amplify, then the pulse frequency may have to be so high that serious power losses occur during the instants of switching, while charge storage in either of the transistor base-emitter junctions can lead to excessive current spikes through the series path then provided by the two transistors. The principle is more readily applicable to small servo-motor systems than to audio amplifiers as the electromechanical properties of the load do not require very high switching frequencies. In some cases efficiencies may exceed $90 \%$ with $100 \%$ as the theoretical upper limit.

In all the above circuits an ever-present problem is that of protecting both the circuit and its load from excessive current flow. Much time and energy is expended on systems for protection against faults, but inevitably accidents happen, so often after some improvement or embellishment has been added. One can only wonder if such thoughts may have been in the mind of William Wordsworth when he wrote
I have submitted to a new control;
A power is gone which nothing can restore;
A deep distress hath humanized my soul.
A cry from the heart that will speak to all designers of power amplifiers.

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